Fall Classic 2005
Individual Test

1) What is $\frac{2}{5}$ of $\frac{1}{2}$?

a) $\frac{4}{5}$ b) $\frac{1}{10}$ c) $\frac{9}{10}$ d) $\frac{3}{7}$ e) NOTA

Problems 2-4 refer to following information:
Kenny has a basket of fruit that contains 3 bananas, 4 oranges and 5 apples.

2) What is the probability of randomly selecting a piece of fruit from Kenny's basket and having it be a banana?

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{5}{12}$ d) $\frac{1}{2}$ e) NOTA

3) What is the probability of randomly selecting two pieces of fruit, without replacement, from Kenny's basket such that both are oranges?

a) $\frac{5}{12}$ b) $\frac{2}{11}$ c) $\frac{1}{11}$ d) $\frac{1}{9}$ e) NOTA

4) What is the probability of randomly selecting three pieces of fruit, without replacement, from Kenny's basket and having all three be different types of fruit?

a) $\frac{5}{144}$ b) $\frac{3}{11}$ c) $\frac{5}{24}$ d) $\frac{3}{22}$ e) NOTA

5) The graph of the line $y = 2 – 2x$ contains points in how many of the quadrants of the Cartesian coordinate plane?

a) 1 b) 2 c) 3 d) 4 e) NOTA

6) How many distinct arrangements of the letters in the word COLORADO are there?

a) 20160 b) 3360 c) 6720 d) 60480 e) NOTA

7) Express the sum as a reduced common fraction:

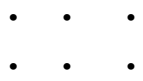
$$.1 + .02 + .003 + .0004 + .00005$$

a) $\frac{12345}{100000}$ b) $\frac{81}{400}$ c) $\frac{93}{500}$ d) $\frac{2469}{20000}$ e) NOTA

8) Chris rudely bumps his friend causing him to spill yogurt on his shirt. In response, his friend proceeds to smear yogurt over the back of Chris' neck. Chris uses his hand to wipe off the yogurt at a speed of 1 wipe for every 2 seconds and removes 10% of the remaining yogurt with each wipe. If Chris is satisfied if at least 50% of the yogurt is removed, how many seconds will he spend removing it?

a) 10 b) 12 c) 14 d) 16 e) NOTA

9) How many equilateral triangles can be formed using the grid below if at least 2 of the vertices of the triangle must be on points of the grid?



a) 24 b) 30 c) 36 d) 42 e) NOTA

10) Find the area of a triangle with vertices on the points (7,2), (2,5), and (2,2).

a) 30 b) 15 c) $\frac{15}{2}$ d) 24 e) NOTA

11) If the wheel of Ari's bike makes 2640 revolutions per hour and has a diameter of 8 feet, how fast, in miles per hour, is Ari going?

a) 8π b) 10π c) 12π d) 14π e) NOTA

12) Wayne, Bryson, and Young are engaged in a Nerf gun game. On a given player's turn, he has the option of shooting once at one of his opponents. If he successfully hits them, they are removed from the game. On any given turn, Wayne has a 20% chance of hitting his target, Bryson has a 70% chance of hitting his target, and Young never misses. If Wayne goes first, then Bryson, then Young, repeating that order until one player remains, and each player uses the best possible strategy to win, then what is the probability that Wayne wins, rounded to the nearest tenth?

a) .1 b) .2 c) .3 d) .4 e) NOTA

13) Which of the following positive two digit numbers has the property that the positive difference of it and the reverse of its digits is equal to the cube of the absolute value of the difference of its tens and units digit?

a) 24 b) 36 c) 26 d) 48 e) NOTA

14) A box contains 8 ceramic eggs, and a shipment contains 14 of these boxes. A chicken farmer orders 4 such shipments in order to boost the morale of his chickens. If each individual egg has a $\frac{1}{10}$ chance of breaking, what is the expected number of unbroken eggs when the farmer receives his order?

a) 44.8 b) 45 c) 403 d) 403.2 e) NOTA

15) What is $(321_5 - 223_6)101_7$ in base 8?

a) –50 b) 50 c) –62 d) 62 e) NOTA

16) If b times c is an integer, which of the following must also be an integer?

a) b b) c c) $\frac{b}{c}$ d) b – c e) NOTA

17) If a three-digit number 34A is divisible by 3, what is the sum of the possible integer values of A?

a) 2 b) 5 c) 12 d) 15 e) NOTA

18) Who was the first person to use *e* to symbolize the base of a natural logarithm?

a) Euler b) Descartes c) Euclid d) Einstein e) NOTA

19) Ed plans to rotate the five tires on his new four-wheel truck so that each tire will be in contact with the road for an equal number of miles. If he drives the truck 40,000 miles, how many miles will each tire be in contact with the road?

a) 40,000 b) 32,000 c) 24,000 d) 8,000 e) NOTA

20) Find the median of the following dataset: $(\pi, \sqrt{10}, \frac{22}{7})$.

a) $\sqrt{10}$ b) $\frac{22}{7}$ c) π d) $\frac{\pi + \frac{22}{7} + \sqrt{10}}{3}$ e) NOTA

21) For how many nonnegative integers n is $\sum_{i=0}^{n} 100^i$ a prime number?

a) 1 b) 2 c) ∞ d) 0 e) NOTA

22) Simplify $7 + 7 \times 7 - 7 \div 7$

a) 13 b) 7 c) 77 d) 97 e) NOTA

23) How many positive integral factors does 667 have?

a) 2 b) 4 c) 6 d) 8 e) NOTA

24) What is the measure, in degrees, of each exterior angle of a regular 18-sided polygon?

a) 10 b) 15 c) 5 d) 20 e) NOTA

25) What is the semi-perimeter of a triangle with side lengths 763, 409, and 1007?

a) 1089.5 b) 1090.5 c) 1091.5 d) 1092.5 e) NOTA

26) Matt is playing a computer game. The longer he plays, the more involved he becomes. The level of his involvement is defined by the function $f(t) = 4t^3 + 5t + 1$, where t is in hours since he started playing. What will the level of Matt's involvement be after 240 minutes?

a) 10 b) 55297201 c) 37 d) 277 e) NOTA

27) If the half-life of a certain substance is ten days, how long will it take for the substance to be reduced to $\frac{1}{4096}$ of its original mass?

a) 100 days b) 90 days c) 11 days d) 110 days e) NOTA

28) How many distinct zeroes does the function $f(x) = x^4 + 4x^3 - 31x^2 - 46x + 168$ have in the interval of $-4 < x < 4$?

a) 2 b) 0 c) 4 d) 1 e) NOTA

29) What is the distance from the origin to the vertex of the parabola $f(x) = -x^2 + 6x - 8$?

a) $\sqrt{7}$ b) $2\sqrt{2}$ c) 3 d) $\sqrt{10}$ e) NOTA

30) How many diagonals can be drawn in a regular hexagon?

a) 6 b) 9 c) 12 d) 15 e) NOTA

31) What is twice the length of the space diagonal of a rectangular prism with edge lengths of 1, 2, and 2?

a) 3 b) $3\sqrt{2}$ c) $3\sqrt{3}$ d) 6 e) NOTA

32) Tom is twice the age that Ben was when Ben was 75% of Tom's age, which was 10 years ago. How many years older is Tom than Ben?

a) 5 b) 0 c) 10 d) greater than 10 e) NOTA

33) The side length of an equilateral triangle with area $\frac{\sqrt{3}}{3}$ is equal to the diameter of a sphere. Find the surface area of that sphere.

a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) π d) $\frac{4\pi}{3}$ e) NOTA

34) For how many integer values of x is $x^2 - 9 < |i|$, where $i = \sqrt{-1}$?

a) 3 b) 5 c) 7 d) 9 e) NOTA

35) Bricks of a certain wall are assembled such that the number of bricks on any given level is $\frac{a_{n-1}}{2} + 4$, where a_{n-1} denotes the number of bricks on the level beneath. If there are 24 bricks on the bottom level and there are 5 levels in the wall, how many bricks are in the wall?

a) 71 b) 72 c) 73 d) 74 e) NOTA

36) For $y \neq 0$, what values of x satisfy the equation $\sqrt{x^2 y - 2y + 4} - 2 = 0$?

a) $\sqrt{2}$ b) 1 c) 0, 1 d) $\frac{\sqrt{2}}{2}$ e) NOTA

37) A clock at the Naval Academy is built such that its hour hand is 7 cm long and its minute hand is 24 cm long. At 3:00 AM, what is the distance, in centimeters, between the tips of the minute and hour hands?

a) $\sqrt{605}$ b) 25 c) $4\sqrt{78}$ d) 168 e) NOTA

38) This same clock runs until 3:20 AM. What is the length of the arc, in centimeters, that the tip of the minute hand has traveled since 3:00 AM?

a) 8π b) 16π c) 48π d) 168 e) NOTA

39) What is the next term in the recursively defined sequence: 2, 4, 16, 49, 169, _?

a) 196 b) 225 c) 256 d) 289 e) NOTA

40) What is the sum of the infinite geometric series with a first term and common ratio of $\frac{1}{1000}$? (Given the formula $a_n = \frac{a_1}{1-r}$, where a_1 is the first term, r is the common ratio, and a_n is your answer)

a) 1 b) $\frac{1}{1001}$ c) $\frac{1}{999}$ d) It diverges e) NOTA

41) Order from greatest to least: $p = \frac{3^{11}}{729}$, $q = \frac{2^{17}}{512}$, $r = \frac{6^8}{7776}$

a) q, p, r b) p, q, r c) q, r, p d) p, r, q e) NOTA

42) What is the volume of the region enclosed by: $x^2 + y^2 + z^2 = x + y + z$?

a) $\frac{\sqrt{3}\pi}{2}$ b) $\frac{9\pi}{16}$ c) $\sqrt{5}$ d) $2\sqrt{2}$ e) NOTA

43) Kate can prepare 5 meals in 3 minutes, Lindsey can prepare 8 meals in 5 minutes, and Jacob can prepare 11 meals in 7 minutes. If the manager can only assign two workers at any given time, what is the shortest amount of time (in minutes) in which this restaurant can prepare 10 meals?

a) 3 b) $\frac{150}{49}$ c) 5 d) $\frac{44}{15}$ e) NOTA

44) Find the sum of the coefficients when $(1 - x + x^2)^{2005}$ is expanded in powers of x.

a) -1 b) 0 c) 1 d) 3^{2005} e) NOTA

45) Define the operation $a \oplus b$ to be equivalent to $\left(\frac{ab}{a+b}\right)^2$. To the nearest tenth, what is $13 \oplus \pi$?

a) 6.4 b) 6.5 c) 6.6 d) 6.7 e) NOTA

46) A game is played in which n players sit around a table and each possesses one penny. Play proceeds in a clockwise fashion. The first player passes one penny to the left, then the next player passes two pennies to the left, the next player passes one penny to the left, the next player passes two pennies to the left and so forth. Once a player possesses no more pennies he is removed from the game. If play continues indefinitely, what is the smallest number n for which one player will NOT end up with all the pennies?

a) 5 b) 6 c) 7 d) 8 e) NOTA

47) Given that $\angle MON = 20^\circ$ and A is a point on $\overline{OM}$ such that OA = $4\sqrt{3}$. If D is a point on $\overline{ON}$, with OD = $8\sqrt{3}$, and B and C are arbitrary points on $\overline{OD}$ and $\overline{AM}$ respectively, find the minimum distance $|AB|+|BC|+|CD|$.

a) 10 b) 12 c) 14 d) 16 e) NOTA

48) How many paths are there from the point (1,2,3) to the point (6,5,4) if each path must consist of a sequence of unit length movements in the positive direction parallel to any of the three axes, and no path can travel through the point (4,4,4)?

a) 318 b) 320 c) 322 d) 324 e) NOTA

49) Find the total number of solutions to: x + y + z + w = 2004 where x, y, z, w are positive odd numbers.

a) 2004 b) $\binom{2003}{3}$ c) $\binom{2004}{4}$ d) $\binom{2007}{3}$ e) NOTA

50) Let x, y, z be positive real numbers satisfying the system:

$$x^2 + xy + \frac{y^2}{3} = 25$$
$$\frac{y^2}{3} + z^2 = 9$$
$$z^2 + zx + x^2 = 16$$

Find $xy + 2yz + 3zx$.

a) $24\sqrt{3}$ b) $18\sqrt{3}$ c) $12\sqrt{3}$ d) $6\sqrt{3}$ e) NOTA

1e 1/5
2a
3c
4b
5c
6c
7d
8c
9b
10c
11e 4π
12c
13b
14d
15c
16e
17d
18a
19b
20b
21a
22e 55
23b
24d
25a
26d
27e 120 days
28a
29d
30b
31d
32a
33d
34c
46c
47b
48d
49e $\binom{1003}{3}$
50a

35a

36e $\pm\sqrt{2}$

37b

38b

39c

40c

41a

42a

43b

44c

45a