

# FALL CLASSIC 2006

## INDIVIDUAL TEST

1) How many ordered triples  $(a, b, c)$  are there where  $a, b,$  and  $c$  are all integers less than 4 and  $a, b,$  and  $c$  are the lengths of the sides of a non-degenerate triangle?

- a) 20      b) 15      c) 8      d) 21      e) NOTA

2) Compute the following:  $\frac{2}{3} + \frac{3}{2} \times \left(\frac{2}{3}\right)^2 - \left(\frac{3}{2}\right)^{-2}$

- a)  $\frac{2}{3}$       b)  $\frac{-11}{4}$       c)  $\frac{8}{9}$       d)  $\frac{7}{6}$       e) NOTA

3) What is the equation of a line in slope/intercept form that is perpendicular to  $3x - 2y = 5$  and passes through the origin?

- a)  $y = \frac{2}{3}x$       b)  $y = \frac{3}{2}x$       c)  $y = \frac{-2}{3}x$       d)  $y = \frac{-3}{2}x$       e) NOTA

4) A palindrome is a number that reads the same forwards and backwards like the number 12,321. How many positive 5-digit palindromes are there that are smaller than 12,321?

- a) 22      b) 23      c) 60      d) 61      e) NOTA

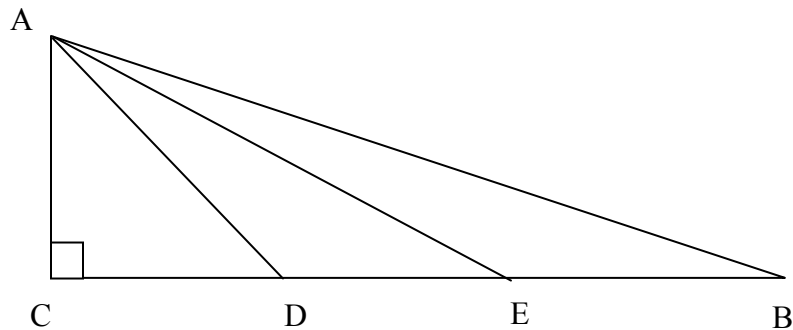
5) If two distinct numbers are selected from the first seven positive prime numbers, what is the probability that their sum is even?

- a)  $\frac{1}{2}$       b)  $\frac{13}{21}$       c)  $\frac{2}{3}$       d)  $\frac{5}{7}$       e) NOTA

6) The geometric mean of two distinct numbers is 12 less than the sum of the two numbers, and 4 less than the difference of the two numbers. What is the product of the two numbers?

- a) 36      b) 16      c) 64      d) 25      e) NOTA

7) In the figure shown below,  $AC = CD = DE = EB$ , and  $AE = 4\sqrt{5}$  inches. What is the number of square inches in the area of triangle  $ADB$ ?



- a) 12      b)  $12\sqrt{5}$       c) 16      d)  $16\sqrt{5}$       e) NOTA

8) If July 1<sup>st</sup> is on a Saturday, what is the sum of all the weekend dates (Saturdays and Sundays) in July?

- a) 10      b) 140      c) 158      d) 172      e) NOTA

9) Thirty-six students took a final exam on which the passing score was 70. The mean of the scores of those who passed was 78, the mean of the scores of those who failed was 60, and the mean of all scores was 71. How many students did not pass the exam?

- a) 12      b) 14      c) 16      d) 22      e) NOTA

10) What is  $65^2 - (53)(57)$ ?

- a) 1,304      b) 1,402      c) 1,204      d) 1,403      e) NOTA

11) Which of the following has the largest area?

- a) an equilateral triangle with side 3cm
- b) a circle with radius 3cm
- c) a square with side 4.5cm
- d) a rectangle with sides 4cm and 5cm

a) a      b) b      c) c      d) d      e) NOTA

12) Add the following: The 2<sup>nd</sup> smallest positive: perfect number, triangular number, perfect square, and perfect cube.

a) 42      b) 36      c) 46      d) 39      e) NOTA

13) The probability that Adam scores at least as well as his rival Jeffrey is  $\frac{3}{5}$ . The probability that Meir scores at least as well as his rival Justin is  $\frac{4}{7}$ . Given that they are independent events, what is the probability that Adam and Meir each get lower scores than their respective rivals?

a)  $\frac{6}{35}$       b)  $\frac{8}{35}$       c)  $\frac{12}{35}$       d)  $\frac{4}{35}$       e) NOTA

14) Rank in order from largest to smallest.

$$A = 3^{100} \quad B = 6^{50} \quad C = 4^{75} \quad D = 2^{125}$$

a) A,B,C,D    b) A,C,B,D    c) C,A,D,B    d) C,B,D,A    e) NOTA

15) Leonardo Pisano- a man of many names: Leonardo of Pisa, Leonardo Biggollo, and Fibonacci, was foremost among medieval Europe's early mathematicians. His contributions included the introduction of algorism to Europe, numerous results in number theory (including studies in perfect numbers and Pythagorean triples), dealings with irrationals, and, of course, that sequence which bears his name and which is today his most well remembered achievement. In terms of which animals did Fibonacci introduce his sequence?

a) cats      b) frogs      c) rabbits      d) flies      e) NOTA

16) The geometric mean of  $n$  numbers is the  $n$ th root of their product. What is the geometric mean of 64, 512, 8, and 384?

- a) 64      b)  $64\sqrt{6}$       c)  $64\sqrt[3]{6}$       d) 128      e) NOTA

17) What is the sum of  $123_4$  and  $1234_5$  as a base 10 number?

- a) 221      b) 222      c) 223      d) 224      e) NOTA

18) A thief stole, without knowing it, the car of the chief of police. The police immediately started an investigation and on the basis of witness depositions, four suspects were arrested that were seen near the car at the time of the crime. Because the chief of police took the case very seriously, he decided to examine the suspects personally and use the new lie detector of the police station. Each suspect gave three statements during the examinations, which are listed below:

Suspect A:

1. In high school I was in the same class as suspect C.
2. Suspect B has no driving license.
3. The thief didn't know that it was the car of the chief of police.

Suspect B:

1. Suspect C is the guilty one.
2. Suspect A is not guilty.
3. I never sat behind the wheel of a car.

Suspect C:

1. I never met suspect A until today.
2. Suspect B is innocent.
3. Suspect D is the guilty one.

Suspect D:

1. Suspect C is innocent.
2. I didn't do it.
3. Suspect A is the guilty one.

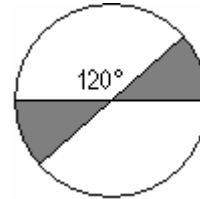
With so many contradicting statements, the chief of police lost track. To make things worse, it appeared that the lie detector didn't quite work yet as it should, because the machine only reported that exactly four of the twelve statements were true, but not which ones. Which suspect is the thief?

- a) A      b) B      c) C      d) D      E) NOTA

19) What is the area of a triangle with vertices at (1,1), (8,1), and (6,5)?

- a) 14      b)  $10\sqrt{2}$       c)  $10\sqrt{7}$       d) 20      e) NOTA

20) In terms of  $\pi$ , what is the number of square inches in the area of the shaded region formed by the intersection of two diameters of length 6 inches in the circle shown?



- a)  $3\pi$       b)  $4\pi$       c)  $6\pi$       d)  $12\pi$       e) NOTA

21) A spigot can fill an empty swimming pool in 12 hours. A second can fill the same empty pool in 18 hours. A drain can empty the full pool in 30 hours. How many hours will it take to fill the pool if all three are open?

- a) 9      b)  $\frac{19}{2}$       c) 10      d)  $\frac{45}{4}$       e) NOTA

22) John and Jane are riding their bikes to school. Suddenly, Jane's bike suffers a flat tire. They decide that John will walk while Jane rides John's bike partway. Jane will then leave the bike by the side of the road and walk the rest of the way. When John gets to the bike, he will ride it the rest of the way. If riding speed is 8 mph, and walking speed is 2 mph, and the remaining distance to the school is 2 miles, how many miles away from the school should Jane leave the bike so that she and John arrive at the school at the same time?

- a)  $\frac{1}{4}$       b)  $\frac{1}{2}$       c)  $\frac{2}{3}$       d) 1      e) NOTA

23) The least integer in a set of consecutive even integers is  $-30$ . If the sum of these integers is 66, how many integers are in the set?

- a) 12      b) 14      c) 30      d) 33      e) NOTA

24) A supermarket buys cartons of orange juice for  $k$  dollars each, and sells them for  $\frac{4k}{3}$  dollars each. How many cartons do they need to sell to make a profit of \$2000?

- a)  $\frac{2000}{k}$     b)  $\frac{6000}{k}$     c)  $\frac{k}{2000}$     d)  $\frac{k}{6000}$     e) NOTA

25) If  $0 < x + y$  and  $y < 0$ , which of the following statements are true?

1.  $x < 0$                       2.  $x < -y$                       3.  $0 < x - y$

- a) 1 only    b) 2 & 3    c) 3 only    d) 1 & 2    e) NOTA

26) If the  $P(A) = .6$  and the  $P(B) = .8$  and they are independent events find  $P(A|B)$ .

- a) .12    b) .32    c) .48    d) .6    e) NOTA

27) How many ways can you arrange the letters in "TAMPABAY"?

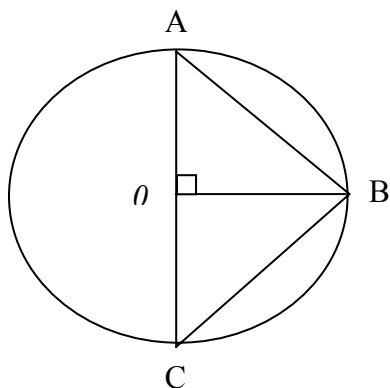
- a) 6720    b) 20220    c) 13480    d) 40440    e) NOTA

28) What is the area of a right triangle if the side opposite angle A (which has a measure of  $60^\circ$ ) is 6cm?

- a)  $\frac{9\sqrt{3}}{2}$     b)  $6\sqrt{3}$     c)  $9\sqrt{3}$     d)  $18\sqrt{3}$     e) NOTA

29) In the figure below,  $O$  is the center of the circle. If  $AB = 6\sqrt{2}$ , what is the area of triangle ABC?

- a)  $36\sqrt{2}$     b) 36    c)  $72\sqrt{2}$     d) 72    e) NOTA



30) What is the standard deviation of the data {1, 2, 3, 4, 5}? (Presume the data represents the population)

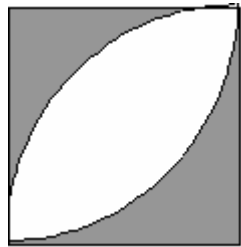
- a) 0      b) 1      c)  $\sqrt{2}$       d) 2      e) NOTA

31) Find the value of x in the following equation:

$$1 + 2 + 3 + 4 + 5 \dots\dots\dots + x = 3,240$$

- a) 60      b) 72      c) 84      d) 88      e) NOTA

32) In the diagram below, each curve is an arc of a circle having center at a vertex of the square with edge 4cm. What percent of the square is shaded? Express your answer to the nearest whole number.



- a) 41      b) 42      c) 43      d) 44      e) NOTA

33) In the multiplication shown each letter represents a digit (not necessarily distinct). What is the sum of all possible products?

$$2A \times B7 = 2CD1$$

- a) 4232      b) 4242      c) 6352      d) 6382      e) NOTA

34) What year is it 3,700 days after December 1<sup>st</sup>, 1995?

- a) 2005      b) 2006      c) 2007      d) 2008      e) NOTA

35) Find the sum of all integers x such that  $2000 < 5^{x-1} < 20000$ .

- a) 13      b) 14      c) 16      d) 18      e) NOTA

36) Using pennies, nickels, dimes and quarters, in how many different ways can you have 30 cents in change?

- a) 15      b) 16      c) 17      d) 18      e) NOTA

37) If the graph  $y = \sqrt{6x - x^2}$  is rotated  $360^\circ$  about the x-axis, what is the volume of the solid that is generated by the rotation?

- a)  $6\pi$       b)  $12\pi$       c)  $18\pi$       d)  $24\pi$       e) NOTA

38) If  $a > b$  and  $c > d$ , which of the following must be true:

- a)  $ac > bd$     b)  $a + b > c + d$     c)  $a - b > c - d$     d)  $ad > bc$     e) NOTA

39) Simplify the following  $\frac{(n+2)! - (n+1)!}{n!}$

- a)  $(n+2)!$     b)  $(n+1)!$     c)  $(n+2)^2$     d)  $(n+1)^2$     e) NOTA

40) A new computer can perform  $x$  calculations in  $y$  seconds and an older computer can perform  $r$  calculations in  $s$  minutes. If these two computers are working simultaneously, how many calculations can be performed in  $t$  minutes?

- a)  $t\left(\frac{x}{60y} + \frac{r}{s}\right)$     b)  $t\left(\frac{60x}{y} + \frac{r}{s}\right)$     c)  $t\left(\frac{x}{y} + \frac{r}{60s}\right)$     d)  $t\left(\frac{x}{y} + \frac{60r}{s}\right)$     e) NOTA

41) A basket contains 10 apples, of which 5 are rotten. What is the probability that a person who buys 4 of these apples at random will select none that are rotten?

- a)  $\frac{1}{2}$       b)  $\frac{2}{5}$       c)  $\frac{2}{25}$       d)  $\frac{1}{42}$       e) NOTA

42) If  $f(x) = x^{\sqrt{x}}$ , then  $f(\sqrt{2}) =$  what rounded to the nearest tenth?

- a) 1.4      b) 1.5      c) 1.6      d) 1.7      e) NOTA

43) If the range of the set of data  $\{1, 1, 2, 2, 3, 3, 3, x\}$  is equal to the mean and  $x$  is an integer, then  $x$  must be:

- a)  $-2$       b)  $-1$       c)  $0$       d)  $1$       e) NOTA

44) The set of points in space equidistant from two distinct fixed points is?

- a) an ellipse   b) a hyperbola   c) a plane   d) a line   e) NOTA

45) In how many ways can a committee of four be selected from nine men so as always to include a particular man?

- a) 84      b) 70      c) 48      d) 56      e) NOTA

46) The integer  $n$  is the smallest positive multiple of 15 such that every digit of  $n$  is either 0 or 8. Compute  $\frac{n}{15}$

- a) 592      b) 587.2      c) 539.2      d) 612      e) NOTA

47) In tetrahedron ABCD, edge AB has length of 3cm. The area of face ABC is  $15\text{cm}^2$  and the area of face ABD is  $12\text{cm}^2$ . These two faces meet each other at a  $30^\circ$  angle. Find the volume of the tetrahedron in  $\text{cm}^3$ .

- a) 30      b) 18      c) 24      d) 28      e) NOTA

48) A mail carrier delivers mail to the nineteen houses on the east side of Elm Street. The carrier notices that no two adjacent houses ever get mail on the same day, but that there are never more than two houses in a row that get no mail on the same day. How many different patterns of mail delivery are possible?

- a) 421      b) 386      c) 483      d) 363      e) NOTA

49) Nine tiles are numbered 1, 2, 3, .....9, respectively. Each of the three players randomly selects and keeps three of the tiles, and sums those three values. The probability that all three of the players obtain an odd sum is  $\frac{m}{n}$ , where m and n are relatively prime positive integers. Find m + n

- a) 13      b) 17      c) 19      d) 21      e) NOTA

50) How many different  $4 \times 4$  arrays whose entries are all 1's and -1's have the property that the sum of the entries in each row is 0 and the sum of the entries in each column is 0?

- a) 70      b) 78      c) 82      d) 90      e) NOTA

1b  
2c  
3c  
4b  
5b  
6c  
7c  
8e  
9b  
10c  
11b  
12e  
13a  
14b  
15c  
16e  
17a  
18b  
19a  
20a  
21e  
22d  
23d  
24b  
25c  
26d  
27a  
28b  
29b  
30c  
31e  
32c  
33a  
34b  
35a  
36d  
37e  
38e  
39d  
40b

41d  
42b  
43d  
44c  
45d  
46a  
47e  
48e  
49b  
50d