Fall Classic 2006
Team Test

1) On the planet Flatterra, there is no global curvature or atmosphere, and there is a constant gravitational pull of 25 $\frac{m}{s^2}$ until you are 20 kilometers from its surface at which point its gravity has no effect on anything. If a rocket is to be launched at an angle of 45 degrees, what is the minimum initial velocity (in kmph) required to escape Flatterra's gravitational pull?

a) 1000 b) 2000 c) 3600 d) 7200 e) NOTA

2) How many of the first 30 positive integers can be expressed as the sum of two distinct prime numbers?

a) 18 b) 19 c) 20 d) 21 e) NOTA

3) Six straws of length 2cm, 3cm, 4cm, 5cm, 6cm, and 7cm, are placed in a box. As a common fraction, find the probability that three straws drawn at random without replacement form a triangle when connected at their endpoints.

a) $\frac{13}{20}$ b) $\frac{11}{24}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$ e) NOTA

4) A rectangular prism has its vertices labeled A, B, C, D, E, F, G, and H. Point A is adjacent to E and F. Point F is adjacent to C. Point D is opposite point C and adjacent to B and H. Point H is adjacent to point E. The length of line segment GH is equal to 3 raised to the power of the length of segment AD. The length of segment FC is equal to twice the length of segment BD minus 6 times the length of EH. If segment BG is 6 units long, what is the length of the line connecting points C and D?

a) 2 b) $2\sqrt{2}$ c) 4 d) $4\sqrt{2}$ e) NOTA

5) How many times must you flip a fair coin so that the probability of having at least one string of at least 4 heads in a row occur is greater than $\frac{1}{4}$?

a) 9 b) 10 c) 11 d) 12 e) NOTA

6) What is the product of all possible solutions to the equation $(x+3)^2=(2x+1)^2$?

a) $-\frac{3}{4}$ b) $-\frac{8}{3}$ c) $\frac{2}{3}$ d)12 e) NOTA

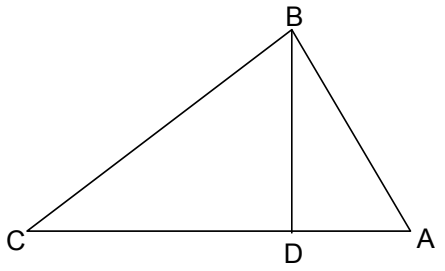
7) In how many zeroes does $(6!)!$ end?

a) 121 b) 152 c) 171 d) 192 e) NOTA

8) 1 is the smallest positive integer that is both a triangular number and a perfect square. What is the third smallest positive integer that is both a triangular number and a perfect square?

a) 1024 b) 1089 c) 1156 d) 1225 e) NOTA

9) Given that $\angle ADB \cong \angle CBA$, $AD = 4$ and $CD = 8$, how many units are in the length of $\overline{AB}$? Express your answer as a decimal rounded to the nearest tenth.



a) 4.8 b) 5.7 c) 6.4 d) 6.9 e) NOTA

10) The sum of the reciprocals of the first four consecutive positive integers is greater than two. What is the least number of consecutive positive integers necessary to make the sum of the reciprocals greater than three?

a) 11 b) 10 c) 9 d) 8 e) NOTA

11) A game is played in which two players take turns removing stones from a pile. On any given turn, one can remove 2, 3, or 4 stones. The loser of the game is whoever takes the last stone (Notice that if the pile ever ends up with a single stone remaining, the game is a draw). Assuming each player plays using their optimal strategy, what will be the outcome of a game starting with 2^{1337} stones?

a) First player wins b) Second player wins c) It results in a draw
d) There is no optimal strategy e) NOTA

12) A wizard decides to reorganize a shelf of his magical tomes. All eight tomes are normally distinguishable from each other. The tomes come in 2 different categories, with four of each category in his collection. If in a given arrangement, two tomes of the same category are placed next to each other, they are indistinguishable from each other and both look exactly how the one farthest to the left would normally appears. However, if they are separated by one or more tomes of a different variety, then each separated set is easily distinguishable from one other. In addition, tomes of different categories are always distinguishable. How many distinguishable arrangements of the eight tomes are there on his shelf?

a) 28,256 b) 40,320 c) 24,872 d) 33,540 e) NOTA

13) Find x, if $\begin{vmatrix} 0 & 2 & 0 & 0 \\ 1 & -1 & 1 & 1 \\ -2 & 2 & 1 & -1 \\ -x & -2 & 0 & -1 \end{vmatrix} = 2$

a) $x = -1$ b) $x = 0$ c) $x = 1$ d) $x = 2$ e) NOTA

14) What is the largest possible four-digit multiple of six, three of whose digits are 2, 3, and 5.

a) 5832 b) 5332 c) 6532 d) 5632 e) NOTA

15) Which of the following is equivalent to: $\prod_{k=1}^{n} \frac{2k-1}{2k}$

a) $\frac{2(n-1)!}{2(n)!}$ b) $\frac{2(n-1)!}{2^n((n-1)!)^2}$ c) $\frac{(2n-1)!}{2^n(2n)!}$ d) $\frac{(2n-1)!}{2^{2n}(2n-2)!}$ e) NOTA

16) Let n be a 6-digit integer such that:

n = abcdef
2n = cdefab
3n = bcdefa
4n = efabcd
5n = fabcde
6n = defabc

where a,b,c,d,e, and f denote the digits of each number. Find a+b+c+d+e+f.

a) 25 b) 26 c) 27 d) 28 e) NOTA

17) Which of the following is odd for all integer n?

a) $3n^2+1$ b) $64n^3+14n-39$ c) (n+1)(n+1)! d) n^3+n-2 **e)** NOTA

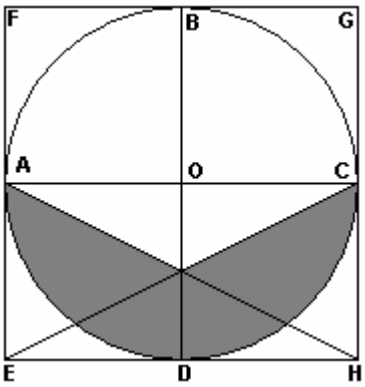
18) A certain three-dimensional figure has 30 edges and 15 vertices. How many faces does it have?

a) 12 b) 13 c) 14 d) 15 e) NOTA

19) In the British card game of Brag, each player is dealt three cards from a standard 52 card deck. The best hand one can achieve is having all three cards of the same rank, i.e. AAA or 333, and this is referred to as a "prial" (short for "pair royal"). If four people are playing in a given hand, what is the probability that at least two of them are dealt a prial?

a) $\frac{1}{978775}$ b) $\frac{6}{978775}$ c) $\frac{36}{978775}$ d) $\frac{216}{978775}$ e) NOTA

20) In the diagram, Circle O is inscribed in the square EFGH with sides of length 12. The diameters AC and BD of Circle O are perpendicular bisectors of EF and FG respectively. What is the area of the unshaded portion inside the outer square? Express your answer in terms of π.



a) $162-18\pi$ b) $144-16\pi$ c) $108-24\pi$ d) $96-30\pi$ e) NOTA

1. E $3600\sqrt{2}$
2. C (20)
3. A
4. E (11)
5. C
6. B
7. E
8. D
9. D
10. A
11. B
12. A
13. C
14. E 8532
15. E $\dfrac{(2k-1)!}{2^{2k-1}(k-1)!(k)!}$
16. C
17. B
18. E
19. C
20. A