Skyview Invitation Games for Mathematical Achievement - 2005

INDIVIDUAL TEST

Directions: Multiple Choice. Choose the BEST answer. You will have 50 minutes for 53 questions.

1: A 30 ft rope is cut into 5 equal pieces. How many cuts were made?

A) 6 B) 5 C) 4 D) 3 E) NOTA

2: If $x\log_{25} x = \log_5 2$ then x equals:

A) 3.5 B) 3 C) 2.5 D) 2 E) NOTA

3: The point P(4, 1) lies inside, outside, or on the circle $x^2 - 6x + y^2 + 2y = -1$ or is it the center of the circle?

A) Center B) Inside C) Outisde D) On E) NOTA

4: A sequence is defined recursively as $a_1 = -4$, $a_2 = 2$, and $a_n = a_{n-1} + 2a_{n-2}$. Find the value of a_5

A) -14 B) -8 C) -6 D) -2 E) NOTA

5: If $2x - y = 7$ and $3y - x = -14$, then $2y + x$ equals:

A) -12 B) -11 C) 8 D) 10 E) NOTA

6: If $|3x + 2| = 16$ and $4x - 3 \le 5$ then x equals:

A) 5 B) 1 C) -3 D) -6 E) NOTA

7: If $x^2 + y^2 = 4$ then the number of ordered pairs (x, y) where x and y are integers is:

A) 1 B) 2 C) 4 D) 6 E) NOTA

8: Determine the number of integral values that a side of a triangle can have if the other two sides are 3 and 10.

A) 7 B) 5 C) 4 D) 3 E) NOTA

9: The number of solutions in positive integers of $2y + 3x = 19$ is

A) 7 B) 6 C) 5 D) 4 E) NOTA

10: Sam is fourteen years older than Joe. Seven years ago, Sam was one year older than twice Joe's age. How old is Joe now?

A) 17 B) 19 C) 21 D) 23 E) NOTA

11: Find the distance from the point (4, 7) to the line that passes through the points (5, 12) and (0, 0).

A) 4 B) 3 C) 2 D) 1 E) NOTA

12: Suppose five ordinary dice are rolled. What is the probability that at least one 6 appears?

A) $1-\left(\frac{5}{6}\right)^5$ B) $\left(\frac{1}{6}\right)^5$ C) $\left(\frac{5}{6}\right)^5$ D) $\frac{1}{6}$ E) NOTA

13: Find the area of a circle inscribed in a triangle with sides 5, 7, and 10

A) $\frac{2\sqrt{66}}{11}$ B) $\frac{43}{11}\pi$ C) 22π D) $\frac{24}{11}\pi$ E) NOTA

14: Find the sum of the positive integer divisors of 99

A) 57 B) 152 C) 155 D) 156 E) NOTA

15: If $x+2y=11$ and $3x+y=13$, then $x+y=$
A) 3 B) 4 C) 5 D) 6 E) NOTA

16: The sum of the squares of the roots of x^3+2x^2-2x-2 is
A) 8 B) 4 C) 2 D) 0 E) NOTA

17: I have two dice, one red and one blue. When the two dice are rolled, the probability that the number of showing on the red die is larger than the number showing on the blue die is
A)1/2 B) 19/36 C) 2/3 D) 5/12 E) NOTA

18: A consumer group is concerned about the possibility that there is systematic underweighing in the meat department of a local supermarket. A representative is sent to purchase 5 one-pound packages of ground meat. If there are currently 12 such packages on display and 8 of them are actually underweight, what's the probability that exactly 3 of the 5 packages purchased are underweight?
A) 40/243 B) 60/243 C) 80/243 D) 120/243 E) NOTA

19: Cheetahs are successfully 20% of the times they attempt to kill their prey. Waking up particularly hungry one day, a cheetah decides it will take three kills to satisfy her appetite. What is the probability the third kill happens on the fifth attempt?
A) .09 B) .11 C) .06 D) .12 E) NOTA

20: At 2:15 o'clock what is the measure (in degrees) of the angle between the hour and the minute hands of the clock?
A) 22.5 B) 24 C) 23.5 D) 23 E) NOTA

21: What is $43_5 \times 24_5$ in base 5
A) 1233 B) 2233 C) 2133 D) 1333 E) NOTA

22: Find the area of the circle whose graph passes through point (2, 1) and is tangent to the x axis at (3, 0)
A) π B) 2π C) 3π D) 4π E) NOTA

23: How many real solutions does the equation $s=\sqrt{s}-1$ have?
A) 4 B) 2 C) 1 D) 0 E) NOTA

24: What is the unit digit of $2^{19}+7^{99}$?
A) 1 B) 3 C) 5 D) 7 E) NOTA

25: Mary has two different gardens which are both squares. The wire to fence the smaller garden cost $20, while the cost of the wire for the larger garden was $30. She can hoe the entire smaller garden in one hour. How long in minutes should it take her to hoe the larger garden?
A) 60 B) 80 C) 70 D) 90 E) NOTA

26: How many different ways can seven people be seated in a row if two people, Pavan and Thomas, must be seated next to each other?
A) 720 B) 1440 C) 120 D) 980 E) NOTA

27: How many integers between $1 \le n \le 100$ are divisible by 2 and 7?
A) 9 B) 8 C) 12 D) 10 E) NOTA

28: A store buys goods at 40% less than list price. At what percent of list should the goods be marked so that the store can give a discount of 20% off the marked price and still make a profit of 20% of the price the store paid for the goods?

A) 68% B) 72% C) 70% D) 74% E) NOTA

29: A clock strikes the hours, that is if it is 2 o'clock it strikes twice. (Non-military time: 1 pm has 1 strike) How many strikes does it make during 24 hours?

A) 153 B) 154 C) 155 D) 156 E) NOTA

30: Take a number, add 42, and take the positive square root. The result is the original number. What was the original number?

A) 7 B) 7 or -6 C) 2 D) -6 E) NOTA

31: Given 8 red jelly beans, 2 green, 6 black and 10 yellow, find the smallest number of jelly beans you must choose, without looking at the colors while choosing, which guarantees that you have at least 5 of the same color

A) 26 B) 25 C) 22 D) 15 E) NOTA

32: Find the sum of all the coefficients of $(x+y)^5$

A) 64 B) 128 C) 32 D) 16 E) NOTA

33: If two marbles are removed from a bag containing only black and white marbles, the probability that they are both white is 1/3. If three marbles are removed, the probability that they are all white is 1/6. How many marbles of each color are in each bag

A) 4 black 6 white B 6 black 4 white C) 5 black 3 white
D) 3 white 5 black E) NOTA

34: Suppose f(x) is a function with f(n)=f(n-2) for all integers n. If f(10)=2, then f(0)=?

A) -4 B) -2 C) 0 D) 2 E) NOTA

35: Find the unit digit of 3^{100}?

A) 3 B) 9 C) 7 D) 1 E) NOTA

36: Find $\sum_{k=1}^{100} \frac{1}{(n)(n+1)}$

A) 1 B) 101/100 C) 101/99 D) 99/100 E) NOTA

37: If Cos(x)=3/4, find Sin(2x)=?

A) $\frac{2}{3}\sqrt{7}$ B) $\frac{3}{2}\sqrt{7}$ C) $\frac{3}{4}\sqrt{7}$ D) $\frac{4}{3}\sqrt{7}$ E) NOTA

38: If $f(x) = x^2 + 3x + c$ and the least possible value of $f(x)$ is 1, then c=?

A) 1/4 B) 3/2 C) 2 D) 13/4 E) NOTA

39: The average grade of 20 students in a class on a test is 72 points. Pavan and Thomas received 0 points on the test (because they didn't show up). Calculate the average grade of the remaining 18 students.

A) 84 B) 82 C) 80 D) 78 E) NOTA

40: The sum of the measures of the interior angles of a regular polygon is 3240 degrees. What is the degree measure of an exterior angle?

A) 24 B) 12 C) 15 D) 36 E) NOTA

41: What is the smallest integer divisible by 2, 3, 4, 5, 6, 7, 8, 9, and 10?

A) 1890 B) 1260 C) 2700 D) 2100 E) NOTA

42: $\frac{\log 2}{\log 4}+\frac{\log 4}{\log 2}+\frac{\log 4}{\log 16}+\frac{\log 16}{\log 4}$ = ?

A) 4 B) 5 C) 6 D) 7 E) NOTA

43: Let $i=\sqrt{-1}$. If $(a+bi)^{12}=5+12i$, then $(b-ai)^{12}$ = ?

A) $5+12i$ B) $5-12i$ C) $12+5i$ D) $12-5i$ E) NOTA

44: The smallest four digit positive prime number is

A) 1001 B) 1003 C) 1007 D) 1009 E) NOTA

45: Which number below is the greatest

A) 6^{100} B) 5^{200} C) 4^{300} D) 3^{400} E) NOTA

46: Find g(3), where $f(x)=x^2-2x+4$, and g is the inverse of f

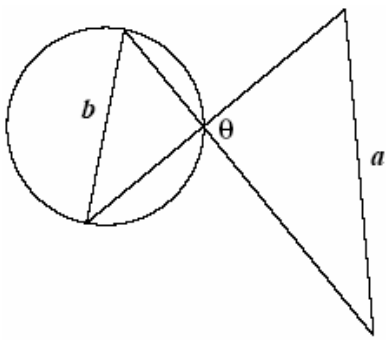
A) 2 B) 1 C. 0 D) -2 E) NOTA

47: Dan can paint a barn in 8 hours. Joe can paint the same house in 4 hours. How many **minutes** will it take them to paint it together?

A) 150 B) 170 C) 160 D) 180 E) NOTA

48: In the figure at right, $\theta=\frac{\pi}{2}$, and that a/b = 3/2, the area of the circle in terms of a is:

A) $\frac{\pi a^2}{9}$ B) $\frac{2\pi a^2}{3}$ C) $\frac{4\pi a^2}{9}$ D) $\frac{\pi a^2}{3}$ E) NOTA



49: How many zeros does 60! end with?

A) 12 B) 13 C) 14 D) 15 E) NOTA

50: What is the sum of $\sum_{n=1}^{20}((n+1)(n+2))$

A) 3540 B) 3580 C) 3600 D) 3620 E) NOTA

51: What is the sum of the infinite series: $\sum_{n=0}^{\infty}\left(n\left(\frac{2}{3}\right)^n\right)$

A) 3 B) 6 C) 9 D) 12 E) NOTA

52: Determine the sum of all values of x: $\log_3 x^4+\log_x 3^{21}=20$

A) $21\sqrt{3}$ B) $27\sqrt{3}$ C) $33\sqrt{3}$ D) $30\sqrt{3}$ E) NOTA

53: If Jack and Joe each flip a fair coin and continues until he has flipped exactly two heads, what's the probability they both stop after the same number of flips

A) 1/2 B) 2/9 C) 4/27 D) 5/27 E) NOTA

INDIVIDUAL
Answer Key

1: C
2: D
3: B
4: A
5: E
6: D
7: C
8: B
9: E
10: E
11: D
12: A
13: D
14: D
15: E
16: A
17: D
18: C
19: E
20: A
21: E
22: A
23: D
24: A
25: E
26: B
27: E
28: E
29: D
30: A
31: D
32: C
33: A
34: D
35: D
36: E
37: E
38: D
39: C
40: E
41: E
42: B
43: A
44: D
45: D
46: B
47: C
48: A
49: C
50: A
51: B
52: D
53: D