

**SIGMA 2005
Team Test**

Multiple Choice. Using a number 2 pencil, indicate the BEST answer on your Scantron.

1. Let $a_1, a_2, a_3 \dots, a_{100}$ be defined as follows:

$$a_{100} = 100$$

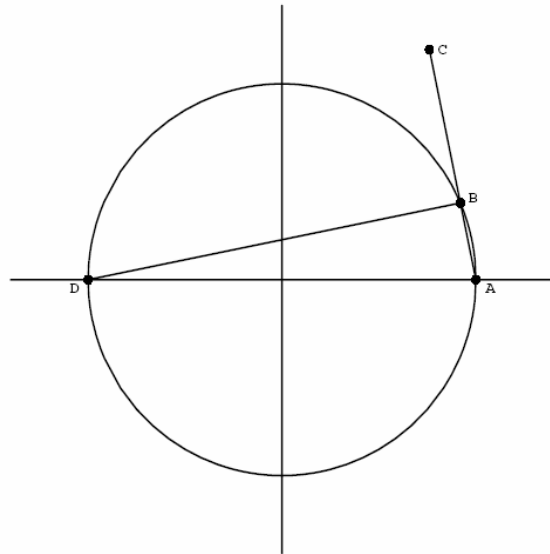
$$a_{99} = 99^{100}$$

$$a_{98} = 98^{99^{100}}$$

Thus, in general $a_n = n^{a_{n+1}}$ for $2 \leq n \leq 99$. Calculate the units digit of a_2 .

- A. 2 B. 4 C. 6 D. 8 E. NOTA

2. In the drawing at right, the equation of the circle is $x^2 + y^2 = 1$. The distance from A(1,0) to B, on the circle, is equal to the reciprocal of the distance from A to C. Calculate the x-coordinate of C. (Note, measurements may not be to scale.)



- A. $\frac{1}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\sqrt{3}$
D. $\frac{2\sqrt{3}}{3}$ E. $\frac{\sqrt{2}}{2}$

3. What is the sum of all the numbers of the form $a*b*c$ where a is from the set $\{1, 2, 4, 8\}$, b is from the set $\{1, 3, 17, 19\}$, and c is from the set $\{1, 7, 31, 61\}$?

- A. 125,000 B. 150,000 C. 175,000
D. 200,000 E. 225,00 F. 60,000

4. Dave and Michael that decide to play the following game. They begin with $S = 0$. They take turns picking a number from the set $\{1, 2, 3, 4, 5, 6\}$. On each turn any of the 6 numbers can be chosen. The number is added to S . The first person who chooses a number which when added to S totals 40 is the winner. Dave begins by picking the number 6. What number is now the best choice for Michael to pick?

- A. 1 B. 3 C. 4 D. 6 E. NOTA

5. A chessboard contains 64 one-by-one squares, but it also contains some two-by-two squares, three-by-three squares, four-by-four squares, and so on. In particular, there is one eight-by-eight square on a chessboard. How many total squares are there on a chessboard?

- A. 200 B. 202 C. 204 D. 206 E. 208

6. A tank with a capacity of 1024 gallons is filled with alcohol. One quarter of it is drawn off and replaced with water, and the mixture is stirred till it is even. How much alcohol in gallons will remain in the tank if this is performed 4 times?

- A. 0 B. 243 C. 324 D. 432 E. NOTA

7. There are 24 different ways that the people attending a party can be seated around a circular table. How many people are at the party?

- A. 4 B. 5 C. 6 D. 7 E. NOTA

8. Find the value of a if $x^3 - 6x^2 + ax + b$ has a triple root.

- A. 7 B. 8 C. 9 D. 10 E. NOTA

9. If both sides of a rectangle R are increased by 1 foot, the area is doubled. If both sides are increased by 3 feet the area is multiplied times 5. Find the area of R .

- A. 5 B. 6 C. 7 D. 8 E. NOTA

10. Given a 3 digit integer n when its digits are reversed and the new number is subtracted from n , the result is a positive integer s . What is the least possible value of s ?

- A. 90 B. 99 C. 100 D. 198 E. NOTA

11. A company produces toys, all of which are painted with exactly two of the three colors black, red, and white. One month they produce 80% with black, 70% with red, and 50% with white. What percentage are both black and white?

- A. 20 B. 25 C. 35 D. 50 E. NOTA

12. Two persons alternatively toss a fair coin until one player first obtains a head and wins the game. If the first player puts up one dollar, what amount of money should the second player put up to make the game fair?

- A. \$0.25 B. \$0.33 C. \$0.50 D. \$0.75 E. NOTA

13. Where defined, $\frac{2x}{x^2-1} - \frac{1}{x-1} =$

- A. $\frac{1}{x-1}$ B. $\frac{1}{x+1}$ C. $\frac{2x-1}{x^2-1}$ D. $x-1$ E. $x+1$

14. If $t = \frac{1}{2}\left(x^3 - \frac{1}{x^3}\right)$, where $x > 0$; then $\sqrt{1+t^2}$ is:

- A. $\frac{1}{2}\left(x^3 - \frac{1}{x^3}\right)$ B. $\frac{1}{2}\left(x^{3/2} - \frac{1}{x^{3/2}}\right)$ C. $\frac{1}{4}\left(x^3 - \frac{1}{x^3}\right)$ D. $\sqrt{\frac{1}{4}x^3 + \frac{3}{2} + \frac{1}{4x^3}}$
E. NOTA

15. Write down the natural numbers in a sequence without spaces or commas: 12345678910111213...9899100 etc.

What is the 10000th digit?

- A. 0 B. 2 C. 8 D. 9 E. NOTA

16. The radius of a cylinder is r and its height is h . It is found that by increasing either its radius or its height by x , its volume is increased by the same amount. Find x .

- A. $\frac{r(r-2h)}{h}$ B. $\frac{r-h}{h}$ C. $\frac{r(r+2h)}{h}$ D. $\frac{r(r-h)}{r+h}$ E. NOTA

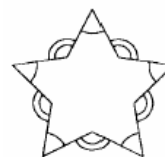
17. On Wednesday, John, in a hurry to get home from work, walked up the escalator while it was moving up. He took one step per second and reached the top in 20 steps. On Thursday, in an even bigger hurry, he climbed at the rate of 2 steps per second and reached the top in 32 steps. On Friday the escalator was stopped for maintenance and he had to walk up the escalator while it was NOT moving. If he took one step at a time, how many steps did he have to take to reach the top?

- A. 48 B. 56 C. 64 D. 80 E. 96

18. Let R_0 be the ray in the first quadrant of the x - y plane with initial point $(0,0)$, and which makes an angle of 55 degrees with the x -axis. Let R_1 and R_2 denote the two rays in the x - y plane with the common initial point $(0, 0)$ so that R_1 lies in the first quadrant, R_2 lies in the second quadrant, the angle between R_0 and R_1 has equal measure as the angle between R_0 and R_2 . If α is the value in degrees of the angle made by R_1 and the positive ray of the x -axis, then the possible range of α is which of the following?

- A. $[0,15)$ B. $[0,20)$ C. $[0,25)$ D. $[0, 30)$ E. $[0,35)$

19. The interior angles of a five pointed star are equal, and so are the concave exterior angles. If the measure of the interior angle of a point is half the measure of a concave exterior angle, what is the measure of an interior angle of a point, in degrees (not to scale).



- A. 60 B. 54 C. 72 D. 144 E. NOTA

20. In an orchestra piece, the violas play a measure every 4 beats and the oboe plays a measure every 3 beats. (The whole orchestra plays beats together). The oboes and the violas all have 6 notes in every measure. How many more notes does the oboe play than a viola in the first 72 beats of the piece?

- A. 6 B. 12 C. 30 D. 36 E. NOTA

21. What is the ratio of a hexagon's area to that of its inscribed circle? Express in terms of π and simplified radicals.

- A. $\frac{2\sqrt{3}}{\pi}$ B. $\frac{\sqrt{3}}{3\pi}$ C. $\frac{\pi\sqrt{3}}{6}$ D. $\frac{\sqrt{3}}{\pi}$ E. NOTA

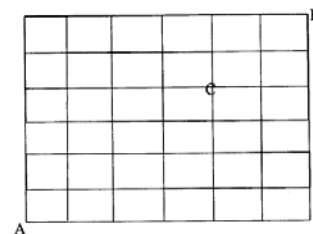
22. The width of a rectangle is x ". The length is 1" less than 3 times the width, and the diagonal is 1" more than 3 times the width. Find the length of the diagonal in inches.

- A. 39 B. 37 C. 35 D. 30 E. NOTA

B

23. In the diagram at right, how many ways can A travel to B (traveling on the line segments), given the restriction that A can go only up or right, and cannot go through c?

- A. 504 B. 714 C. 848 D. 924 E. NOTA



SIGMA 2005
Team Test ANSWERS

1. A
2. A
3. THROWN OUT – Answer is 60,000
4. D
5. C
6. D
7. B
8. E (12)
9. B
10. B
11. E (30)
12. C
13. B
14. E($\frac{1}{2}\left(x^3 + \frac{1}{x^3}\right)$)
15. E (1)
16. B
17. D
18. B
19. C
20. D
21. A
22. B
23. A