

Skyview Invitational Games for Mathematical Achievement  
Speed Math– 2006-2007 ROUND 1

Name \_\_\_\_\_ School \_\_\_\_\_  
Division (circle one)    Mu    Alpha    Theta

Score: \_\_\_\_\_  
Verify: \_\_\_\_\_

Instructions: Write all answers in EXACT form in the box. Anything outside the answer box will be disregarded. Leave all answers in terms of  $\pi$  and reduced radicals. Express all fractions in improper form reduced to lowest terms. Make sure you write your answer in the correct box. You will have 15 minutes.

1. How many distinct positive divisors does  $({}_{20}C_0 + {}_{20}C_1 + {}_{20}C_2 + \dots + {}_{20}C_{20})$  have?
2. What is the sum of the squares of the roots of  $5x^5 - 4x^4 + 3x^3 - 2x^2 + x + 1$ ?
3. The UW was founded in 1861. I was born in 1987, 126 years later. What is the largest prime less than 126?
4. Find the smallest positive integer  $x$ , such that  $x \equiv 7^{20} \pmod{5}$ .
5. What is the largest prime factor of 2006?
6. How many 4 digit numbers have at least one '0'?
7. Let  $N$  be the sum of the odd numbers from 1 to 1001, inclusive. What is  $\sqrt{N}$ ?
8. In  $x = \log_8 2$ , what is  $x^6$ ?
9. The SkyCity restaurant on the Space Needle revolves once every 47 minutes. If you look outside every 30 minutes only, after how many minutes will you see the same view?
10. I inscribe a square in a circle of radius  $\pi$ . What is the area of the square?
11. What is the area of the ellipse defined by  $2x^2 + 16x + 5y^2 + 10y = 13$ ?
12. Kim goes kayaking, paddling once every 3 seconds starting at  $t=0$  (then at  $t=3$ , etc). If each stroke moves her forward 5 ft, and the wind continuously blows her back at 1 ft/sec, how many seconds will it take to cross the finish line 99 ft away (for the first time)?
13. The sum of two real numbers is  $\frac{5}{2}$ , their product is 1. What is the smaller number?
14. A goat is tied to the vertex of a barn shaped like an equilateral triangle with sides of 6 meters. If the goat's rope is 7 meters long, what is the area ( $m^2$ ) of the range it can graze?
15. The point (4,3) is the vertex of the parabola  $ax^2 + bx + 1$ . Find  $a$ .
16. Shinkansen, the Japanese bullet train, travels at 300 km/h. A bullet travels at 200 m/s. How much earlier (minutes) would a bullet arrive than the Shinkansen on a 100 km journey?
17. Find the largest positive integer  $n$  which has the following property: All of the positive integers less than  $n$  that are relatively prime to  $n$  are prime numbers except for 1
18. Find A if  $\frac{12x+10}{x^2-16} = \frac{B}{x-4} + \frac{A}{x+4}$
19. If  $\sqrt{21+\sqrt{320}} = x + \sqrt{y}$ , and  $x$  and  $y$  are positive integers, then  $x + y = ?$
20. The interior angles of a quadrilateral are in arithmetic progression. If the largest angle is five times the smallest, find its smallest angle in degrees;

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