

SIGMA 2006-2007

Team Test

Multiple Choice. Using a number 2 pencil, indicate the BEST answer on your Scantron.

1. What is the unit digit, in base 8, of the following sum:

$$(1_8)! + (2_8)! + (3_8)! + (4_8)! + (5_8)! + (6_8)! + (7_8)! + (10_8)! + \dots + (2007_8)!$$

- A. 1_8 B. 3_8 C. 5_8 D. 7_8 E. NOTA

2. Suppose there are 1000 lockers at Skyview High School. Student 1 opens every locker, then student 2 switches every 2nd locker from open to closed or closed to open, then student 3 switches every 3rd locker, then student 4 switches every 4th locker, ... then student 2007 switches every 2007th locker. How many lockers are open at the end of student 2007's switches?

- A. 304 B. 168 C. 44 D. 31 E. NOTA

3. Kunal has a rabbit named Fermat. Fermat can hop up 1 step or 2 steps at a time. How many different ways can Fermat hop up 15 steps?

- A. 2^{15} B. 987 C. 32768 D. 32767 E. NOTA

4. Find $\sum_{i=0}^n a_i$ where $\frac{F_{100}}{F_{99}} = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{\dots + \frac{1}{a_n}}}}$, where F_k is the k^{th} Fibonacci number and a_i is

a natural number.

- A. 98 B. 99 C. 100 D. 101 E. NOTA

5. How many zeros are before the decimal point of $\frac{100!}{(8!)^{13}}$?

- A. 3 B. 6 C. 11 D. 14 E. NOTA

6. A robotic arm, suspended over a bin of 2007 balls, is programmed to read instructions encoded as a sequence of 0's and 1's. A '0' followed immediately by a '1' in the sequences causes the arm to add a ball to the bin, while a 1 followed by a 0 causes the arm to remove a ball from the bin. A '1' followed by a '1' or a '0' followed by a '0' causes no action by the arm. So, if three numbers x,y,z are fed into the machine in order, it will process xy, then yz. (110 will do nothing, then add a ball)

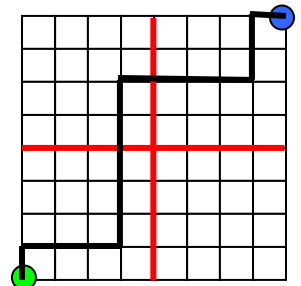
After a single *finite* instruction sequence, what is the largest number of balls that can remain in the bin.

- A. 2007 B. 2008 C. 4014 D. 4015 E. NOTA

7. Find the 2007th term in the sequence $\{1, 2, 3, 5, 8, 11, 12, 13, 15, 18, 21, 22 \dots\}$; the ascending sequence of positive integers ending in one of $\{1,2,3,5,8\}$

- A. 4001 B. 4002 C. 4011 D. 4012 E. NOTA

8. Three consecutive odd integers have a product of 2145. What is their sum?
 A. 39 B. 45 C. 51 D. 57 E. NOTA
9. I have a 2007 digit number, whose digits, when taken 3 at a time, are strictly (no equality) in ascending order (for example, 105106112 has the 3-digit groups 105, 106, and 112, which are strictly in ascending order). How many such numbers exist?
 A. ${}_{2007}C_{900}$ B. ${}_{900}C_{669}$ C. ${}_{900}P_{669}$ D. ${}_{999}C_{669}$ E. NOTA
10. How many ways can you rearrange the letters in CHERNIKOV such that 'I' is directly before 'E', except when they are at some point after C, in which case E must be directly before I { CHEIRNKOV and RIEKNCHOV are such arrangements } ?
 A. $8! * 2$ B. $8! * 3$ C. $\frac{9!}{2}$ D. $2 * 9!$ E. NOTA
11. Let x_y be $x^2 - y^2$; and $w_x y_z$ be the same as $((w_x)_y)_z$. If the letters of the alphabet have values A=1, B=2, ... Z=26; **except E = 0**; and AB indicates A*B:
 What is: (THE)_(MEANING)_(OF)_(LIFE)?
 A. 42 B. 8100 C. 43046200 D. 65610000 E. NOTA
12. A 3x3x3 cube is composed of 27 1x1x1 smaller cubes. 54 faces of these cubes are visible. Suppose 1 of the 26 visible outer cubes is removed at random. What is the expected number of visible small-cube-faces?
 A. $54\frac{1}{13}$ B. $54\frac{8}{9}$ C. $52\frac{11}{13}$ D. $52\frac{8}{9}$ E. NOTA
13. A point is randomly selected at some point within a 2x2x2 cube. A sphere centered at this point is constructed of the smallest possible radius such that it encloses the entire cube. What is the expected radius of sphere? //can you think of a way to solve this without triple integrals?
 //alternatively, you have sphere of radius 2. a point is chosen at random in it. Make a cube centered there, at any orientation, with smallest possible side length. What is the expected/smallest/largest possible side length OR VOLUME
 //easier version: what is the largest possible sphere that could have to be constructed?
 A. B. C. D. E. NOTA
14. A beetle sits at the bottom left corner of an 8x8 checkerboard. It can only travel along the lines of the checkerboard. If it cannot travel on the middle horizontal line or middle vertical line (but is allowed to cross the line, how many ways can it walk to the top right corner of the board moving only up or right.



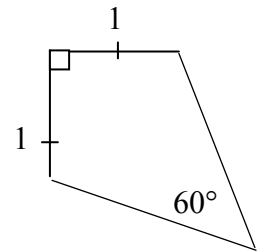
- A. ${}_{14}C_7 - 7!$ B. ${}_{16}C_8 - 7!$ C. ${}_{14}C_7$ D. ${}_{16}C_8$ E. NOTA

15. A duck can swim three times fast as a goose. The duck takes 15 minutes to swim 1 lap. If they are swimming a 1 lap race, how much of a head start should the goose get so the race is a tie?

- A. 5 min B. 10 min C. 15 min D. 30 min E. NOTA

16. What is the area of the kite at right?

- A. $\frac{1+\sqrt{2}}{2}$ B. 1.25 C. $\frac{1+2\sqrt{2}}{2}$ D. $\frac{1+2\sqrt{3}}{2}$ E. NOTA



17. If every god on Mount Math is master of a unique 100 digit power of two, how many gods can there be in the Math pantheon?

- A. 2 B. 3 C. 4 D. 5 E. NOTA

18. A point is chosen at random in a regular hexagon with sides of length 10. A circle of radius 1 is drawn around this point. What is the probability the circle is entirely contained within the hexagon?

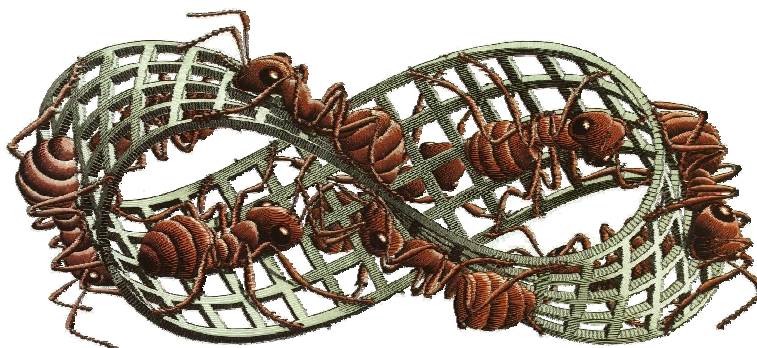
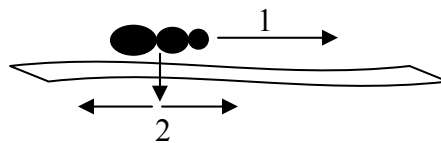
- A. $\frac{76-10\sqrt{3}}{300}$ B. $\frac{9}{10}$ C. $\frac{15-\sqrt{3}}{15}$ D. $\frac{81}{100}$ E. NOTA

19. A proton and an electron walk into a restaurant and order some pi. If the proton can eat a pi in 2 hours, and an electron can eat 2 pi in 3 hours, how long will it take them to finish a pair of pi 's together?

- A. 100 minutes B. $\frac{1}{3}$ hours C. 1.75 hours D. 45 minutes E. NOTA

20. An ant is crawling along a Mobius strip searching for food. Walking straight along the strip, path 1, it takes it 10 minutes to arrive to its starting point. If, alternatively, it is allowed to once switch "sides" on the strip at some point as a shortcut and then continue walking along the strip, path 2, how quickly can it get back to its starting point?

- A. 3 minutes B. 4 minutes C. 5 minutes D. 6 minutes E. NOTA



Ants by M.C. Escher